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Dirac Equation for Spin Density in an Ideal Elastic Solid

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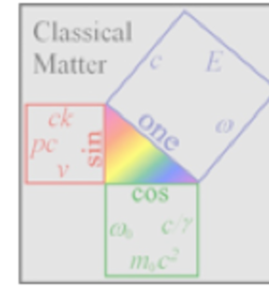
Spin Density



Belinfante-Rosenfeld symmetric stress-energy tensor implies dynamic (wave) & conjugate momenta:

$$\mathbf{P}_{\boldsymbol{Total}} = -\mathrm{Re}[\psi^\dagger i\nabla\psi] + \frac{1}{2}\nabla\times\left[\psi^\dagger\frac{\boldsymbol{\sigma}}{\mathbf{2}}\psi\right]$$

$$= \mathbf{P}_{wave} + \frac{1}{2}\nabla\times\mathbf{s}$$

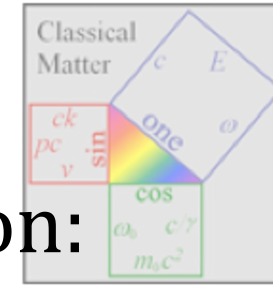
$$\mathbf{J} = -\mathrm{Re}\big(\mathbf{r}\times[\psi^\dagger i\nabla\psi]\big) + \left[\psi^\dagger\frac{\boldsymbol{\sigma}}{\mathbf{2}}\psi\right]$$

$$= \mathbf{L}_{\boldsymbol{wave}} + \mathbf{s}$$

Elastic waves in a solid also have dynamic & conjugate (or “intrinsic”) momenta.

Is Standard Model a decomposition of elastic waves into “particles”?

Classical Spin Density



Incompressible Helmholtz decomposition:

$$\mathbf{p} \equiv \rho \mathbf{u} = \cancel{\mathbf{p}_0 + \nabla \Phi} + \frac{1}{2} \nabla \times \mathbf{s}$$

Angular momentum:

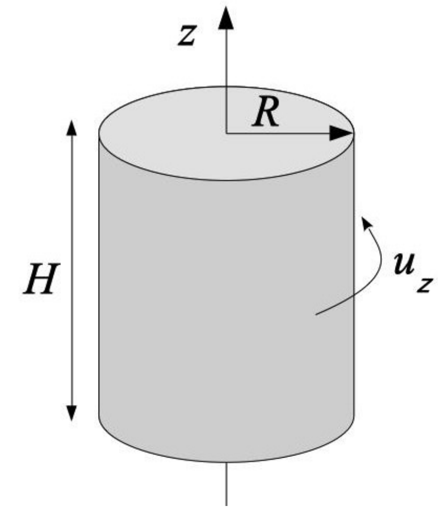
$$\mathbf{S} = \int \mathbf{r} \times \mathbf{p} \, d^3 r = \int \mathbf{s} \, d^3 r \cancel{+ \text{b.t.}}$$

Kinetic energy: $\left(\mathbf{w} = \frac{1}{2} \nabla \times \mathbf{u} \right)$

$$K = \int \frac{1}{2} \rho u^2 \, d^3 r = \int \frac{1}{2} \mathbf{w} \cdot \mathbf{s} \, d^3 r \cancel{+ \text{b.t.}}$$

Spin density is the momentum conjugate to angular velocity ($\mathfrak{L} \sim K$; $\frac{\partial \mathfrak{L}}{\partial \mathbf{w}} = \frac{\mathbf{s}}{\mathbf{2}} + \frac{\mathbf{s}}{\mathbf{2}} = \mathbf{s}$).

Rotating Cylinder



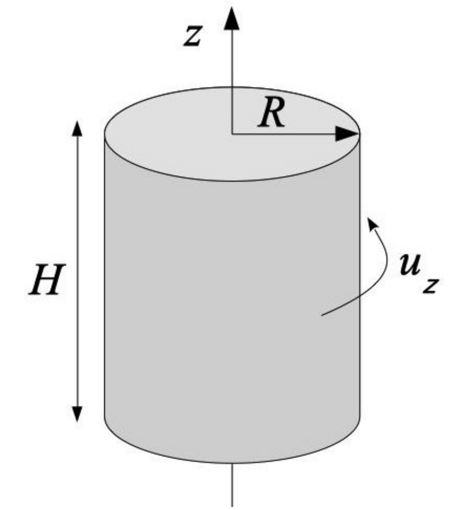
Parabolic radial distribution of spin density.

$$s_z = \rho w_0 [R^2 - r^2] \qquad r \le R$$

$$u_\phi = \frac{1}{2\rho}\frac{\partial}{\partial r} s_z = r w_0 \qquad r \le R$$

$$w_z = \frac{1}{2r}\frac{\partial}{\partial r}(r u_\phi)$$
$$= w_0[1 - R\delta(R - r)/2] \qquad r \le R$$

Rotating Cylinder



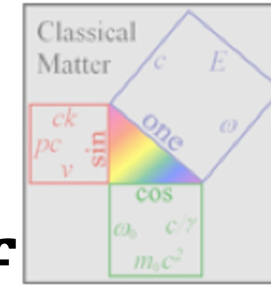
Integration yields the usual total angular momentum and kinetic energy.

$$S_z = \int \rho w_0 [R^2 - r^2]\, d^3r$$

$$= 2\pi\rho H \left(\frac{R^4}{4}\right) w_0 = \frac{MR^2}{2} w_0 = I w_0$$

$$K = \int \frac{1}{2} \mathbf{w} \cdot \mathbf{s}\, d^3r = \frac{1}{2} I w_0^2$$

Equation of Evolution



Momentum density: $\partial_t \mathbf{p} + \mathbf{u} \cdot \nabla \mathbf{p} = \mathbf{f}$

Changes due to convection & force.

Spin density: $\partial_t \mathbf{s} + \mathbf{u} \cdot \nabla \mathbf{s} - \mathbf{w} \times \mathbf{s} = \boldsymbol{\tau}$

Changes ~ convection, rotation, & torque.

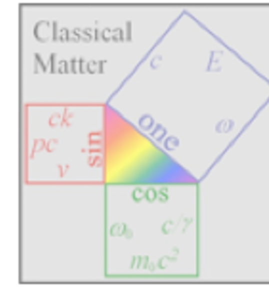
Let $\boldsymbol{s} \equiv \partial_t \mathbf{Q}$, Displacement: $\boldsymbol{\xi} = \frac{1}{2\rho} \nabla \times \mathbf{Q}$

Elastic solid: Torque = $\boldsymbol{\tau} = -\frac{\mu}{\rho} \nabla \times \nabla \times \mathbf{Q}$

$$\partial_t^2 \mathbf{Q} + \mathbf{u} \cdot \nabla \partial_t \mathbf{Q} - \mathbf{w} \times \partial_t \mathbf{Q} = c^2 \nabla^2 \mathbf{Q}$$

Nonlinearity $\Rightarrow$ soliton solutions

Factor the Wave Equation



$$Q(z,t) = Q_F(ct - z) + Q_B(ct + z)$$

- Independent states 180° apart ⇒ spin 1/2
- The second-order wave equations are:

$$\partial_t^2 Q_B - c^2 \partial_z^2 Q_B = 0$$
$$\partial_t^2 Q_F - c^2 \partial_z^2 Q_F = 0$$

- Equivalent to first-order matrix equation:

$$\partial_t \begin{bmatrix} \dot{Q}_B \\ \dot{Q}_F \end{bmatrix} - c \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \partial_z \begin{bmatrix} \dot{Q}_B \\ \dot{Q}_F \end{bmatrix} = 0$$

- The Dirac equation of quantum mechanics simply extends this matrix equation to vectors in 3D.

Factor the Wave Equation

First-order 1-D wave eqn. (chiral $\beta^3 = -\gamma^5$):

$$\partial_t(\psi^T \sigma_{\boldsymbol{z}} \psi) - c\partial_z(\psi^T \beta^3 \psi) = 0$$

$$\partial_t Q = \begin{bmatrix} (\dot{Q}_{B+})^{1/2} \\ (\dot{Q}_{F-})^{1/2} \\ (\dot{Q}_{F+})^{1/2} \\ (\dot{Q}_{B-})^{1/2} \end{bmatrix}^T \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{bmatrix} (\dot{Q}_{B+})^{1/2} \\ (\dot{Q}_{F-})^{1/2} \\ (\dot{Q}_{F+})^{1/2} \\ (\dot{Q}_{B-})^{1/2} \end{bmatrix} = \psi^T \sigma_{\boldsymbol{z}} \psi$$

$$c\partial_z Q = \begin{bmatrix} (\dot{Q}_{B+})^{1/2} \\ (\dot{Q}_{F-})^{1/2} \\ (\dot{Q}_{F+})^{1/2} \\ (\dot{Q}_{B-})^{1/2} \end{bmatrix}^T \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{bmatrix} (\dot{Q}_{B+})^{1/2} \\ (\dot{Q}_{F-})^{1/2} \\ (\dot{Q}_{F+})^{1/2} \\ (\dot{Q}_{B-})^{1/2} \end{bmatrix} = \psi^T \beta^3 \psi$$

Factor the Wave Equation

Matrices for 3-D polarization & wave velocity:

$$\sigma_x = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Factor the Wave Equation

Matrices for 3-D spatial derivatives:

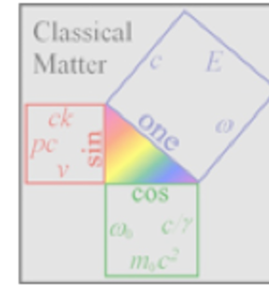
$$\beta^1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \gamma^0 \quad \perp \mathbf{s} \dashrightarrow \text{Curl}$$

$$\beta^2 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} = i\gamma^5\gamma^0 \quad \perp \mathbf{s} \dashrightarrow 0$$

$$\beta^3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = -\gamma^5 \quad || \, \mathbf{s} \dashrightarrow \text{Div}$$

<u>Wave velocity</u> matrices: $-c\beta^3\sigma_i = c\gamma^5\sigma_i$

Linear Dirac Equation



1-D wave equation:

$$0 = \psi^T \sigma_z \{\partial_t \psi + \ c\gamma^5 \sigma_z \partial_z \psi\} + \text{Transp.}$$

$$0 = \partial_t^2 Q_z - c^2 \partial_z^2 Q_z$$

3-D wave equation (replace z with i or j):

$$\psi^\dagger \sigma_i \left[\partial_t \psi + c\,\gamma^5 \sigma_j \partial_j \psi\right] + adj. = 0$$

Same as electron! Mass term drops out.

$$\psi^\dagger \sigma_i \left[\partial_t \psi + c\,\gamma^5 \sigma_j \partial_j \psi + iM\gamma^0 \psi\right] + adj. = 0$$

Wave function describes spin density:

$$s_i = (1/2)\psi^\dagger \sigma_i \psi$$

Dirac Equation

Linear vector wave equations:

$$0 = \partial_t[\psi^\dagger \boldsymbol{\sigma}\psi] + c\nabla[\psi^\dagger\gamma^5\psi]$$
$$-ic[\nabla\psi^\dagger \times \gamma^5\boldsymbol{\sigma}\psi + \psi^\dagger\gamma^5\boldsymbol{\sigma} \times \nabla\psi]$$

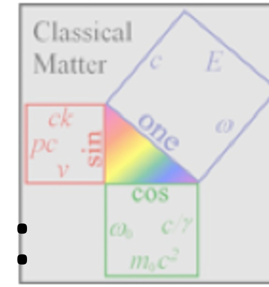
$$0 = \partial_t^2\mathbf{Q} - c^2\nabla(\nabla\cdot\mathbf{Q}) + c^2\nabla\times\nabla\times\mathbf{Q}$$
$$= \partial_t^2\mathbf{Q} - c^2\nabla^2\mathbf{Q}$$

$$\mathbf{s} = \partial_t\mathbf{Q} = (1/2)\psi^\dagger\boldsymbol{\sigma}\psi$$

$$c\nabla\cdot\mathbf{Q} = -(1/2)\psi^\dagger\gamma^5\psi$$

$$c^2\nabla\times\nabla\times\mathbf{Q} = \frac{-ic}{2}[\nabla\psi^\dagger \times \boldsymbol{\sigma}\psi + \psi^\dagger\gamma^5\boldsymbol{\sigma}\times\nabla\psi]$$

Dirac Equation



Nonlinear Dirac eqn. $\boldsymbol{s} \equiv \partial_t \mathbf{Q} = \left[\psi^\dagger \frac{\boldsymbol{\sigma}}{2} \psi\right]$:

$$\partial_t \psi = c\, \beta^3 \boldsymbol{\sigma} \cdot \nabla\psi - \mathbf{u} \cdot \nabla\psi - \frac{\mathrm{i}}{2} \breve{w}_0\, \gamma^0 \psi - \frac{\mathrm{i}}{2} \mathbf{w} \cdot \boldsymbol{\sigma}\psi$$

Propagation, convection, rotation of wave velocity, rotation of the solid medium.

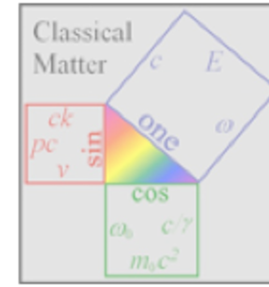
Mass represents rotation of wave velocity.*

Each term has a simple physical interpretation.

For plane waves, velocity rotation cancels rotation of the medium.

*c.f. Hestenes, *Found. Phys.* **20**(10):1213-32,1990

Lagrangian Density



$\mathbf{w}$ & $\breve{w}_0$ terms contribute twice to E-L eqn.

$$\mathfrak{L} = \mathrm{Re}\{\psi^\dagger i\,\partial_t\psi + c\psi^\dagger\gamma^5\boldsymbol{\sigma}\cdot i\nabla\psi + \mathbf{u}\cdot\psi^\dagger i\nabla\psi\} - \frac{1}{4}\breve{w}_0\psi^\dagger\,\gamma^0\psi - \frac{1}{4}\mathbf{w}\cdot\psi^\dagger\boldsymbol{\sigma}\psi$$

$$\mathbf{u} = \frac{1}{2\rho}\nabla\times\ \mathbf{s} = \ \frac{1}{2\rho}\nabla\times\left(\psi^\dagger\frac{\boldsymbol{\sigma}}{2}\psi\right);$$

$$\mathbf{w} = \frac{1}{2}\nabla\times\ \mathbf{u}\ =\ \frac{1}{4\rho}\nabla\times\nabla\times\left(\psi^\dagger\frac{\boldsymbol{\sigma}}{2}\psi\right)$$

$$\breve{w}_0 = \frac{1}{2\rho c}f =\ \frac{1}{4\rho}\nabla^2\left(\psi^\dagger\frac{\gamma^0}{2}\psi\right)$$

Dynamic & Conjugate Momenta

$$\mathbf{P} = -[\psi^\dagger i\nabla\psi] + \frac{1}{2}\nabla \times \left[\psi^\dagger \frac{\boldsymbol{\sigma}}{\mathbf{2}}\psi\right]$$

$$= \mathbf{P}_{wave} + \mathbf{p}_{solid}$$

$$\mathbf{J} = -\mathbf{r} \times [\psi^\dagger i\nabla\psi] + \left[\psi^\dagger \frac{\boldsymbol{\sigma}}{\mathbf{2}}\psi\right]$$

$$= \mathbf{L}_{wave} + \mathbf{s}_{solid}$$

- Momentum consistent with Belinfante-Rosenfeld energy-momentum tensor.
- Wave (orbital) and spin angular momentum.

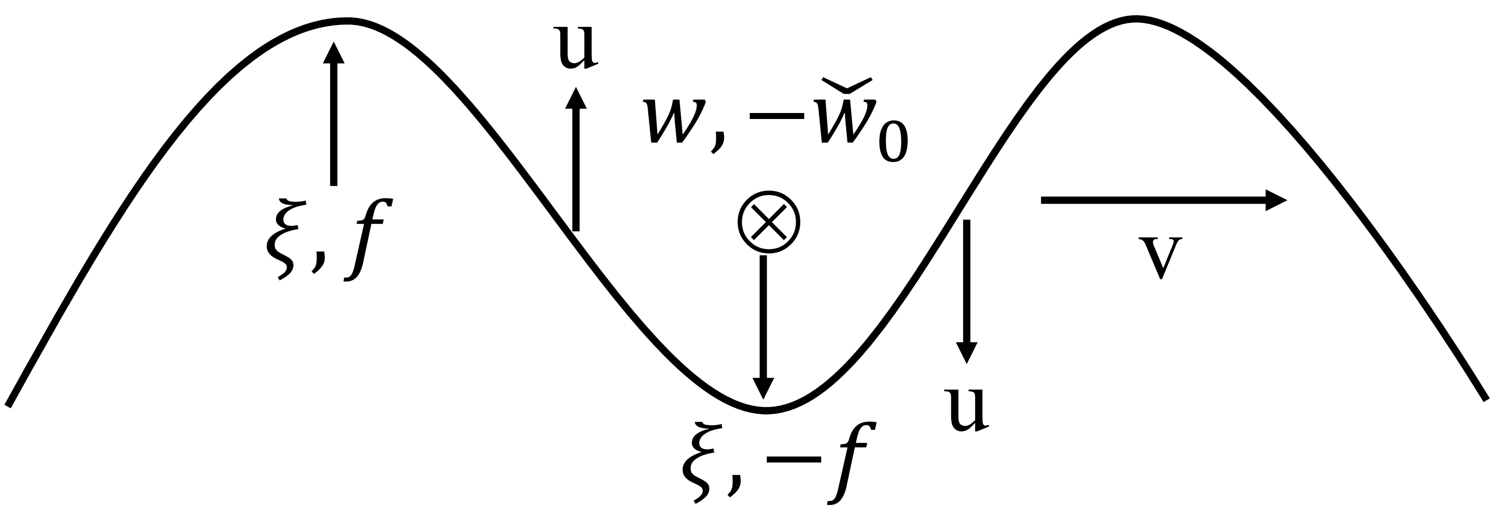
Potential Energy

$\psi^\dagger\gamma^0\psi$ ~ displacement. $\breve{w}_0$ ~ force density:

$$\breve{w}_0 = \frac{1}{2\rho c} f = \frac{1}{4\rho}\nabla^2\left(\psi^\dagger \frac{\gamma^0}{2}\psi\right)$$

PE density is: $U = \frac{\breve{w}_0}{4}\psi^\dagger\gamma^0\psi \sim -\frac{1}{2}\mathbf{f}\cdot\boldsymbol{\xi}$

Rotational PE: $U_R = \mathcal{E} - \mathrm{U}$



Hamiltonian Density

$$\mathcal{H} = \mathrm{Re}\big(-c\psi^\dagger\gamma^5\boldsymbol{\sigma}\cdot i\nabla\psi - \mathbf{u}\cdot\psi^\dagger i\nabla\psi\big) + \frac{\breve{w}_0}{4}\psi^\dagger\gamma^0\psi + \frac{\mathbf{w}}{4}\cdot\psi^\dagger\boldsymbol{\sigma}\psi$$

$$= \left(\mathcal{E} \quad - \quad 0 \quad + \quad \tfrac{1}{2}\mathbf{f}\cdot\boldsymbol{\xi}\right) \quad + \quad \tfrac{1}{2}\mathbf{w}\cdot\mathbf{s}$$

$$= \ U_R + K_R$$

Rotational potential energy: $U_R = \ \mathcal{E} \ + \ \dfrac{1}{2}\mathbf{f}\cdot\boldsymbol{\xi}$

Rotational kinetic energy: $K_R = \frac{1}{2}\mathbf{w}\cdot\mathbf{s} = \mathcal{E} - \frac{1}{2}\rho u^2$

Equal integrals for vectors & bispinors:

Vectors: Wave velocity × wave momentum = $2U$

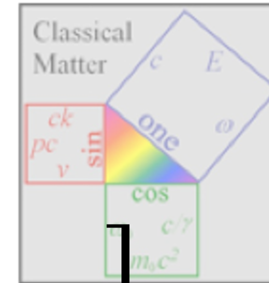
Bispinors: Wave velocity × wave momentum = $\mathcal{E}$

Operators as Generators

$$\mathbf{P}\psi = -i\nabla\psi + \frac{1}{2}\nabla\times\frac{\boldsymbol{\sigma}}{2}\psi$$

$$\mathbf{J}\psi = -\mathbf{r}\times i\nabla\psi + \frac{\boldsymbol{\sigma}}{2}\psi$$

- **J** is generator of rotations (orbital for coordinate, spin for direction)
- **P** is generator of translations.
  - Wave momentum shifts coordinate ($\mathbf{r}$).
  - Intrinsic momentum shifts displacement from equilibrium ($\boldsymbol{\xi}$).



Example: Plane Wave

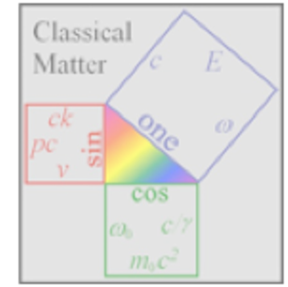
$$\psi = \sqrt{\frac{\omega Q_0}{2}} \begin{bmatrix} 1 \\ \cos(\omega t - kz) - i\,\frac{\omega Q_0 k^2}{4\rho c}\sin(\omega t - kz) \\ \cos(\omega t - kz) - i\,\frac{\omega Q_0 k^2}{4\rho c}\sin(\omega t - kz) \\ 1 \end{bmatrix}$$

$$s_x = \frac{1}{2}\psi^\dagger \sigma_x \psi = \omega Q_0 \cos(\omega t - kz)$$

$$s_y = \frac{1}{2}\psi^\dagger \sigma_y \psi = 0$$

$$s_z = \frac{1}{2}\psi^\dagger \sigma_z \psi = 0$$

Example: Plane Wave



$$\psi = \sqrt{\frac{\omega Q_0}{2}} \begin{bmatrix} 1 \\ \cos(\omega t - kz) - i\frac{\omega Q_0 k^2}{4\rho c}\sin(\omega t - kz) \\ \cos(\omega t - kz) - i\frac{\omega Q_0 k^2}{4\rho c}\sin(\omega t - kz) \\ 1 \end{bmatrix}$$

$$U_R = \left(-\mathrm{Re}\left(\psi^\dagger \gamma^5 \boldsymbol{\sigma} \cdot i\nabla\psi\right) + \frac{1}{4}\breve{w}_0 \psi^\dagger \gamma^0 \psi\right)$$

$$= \frac{k^2\omega^2 Q_0^2}{8\rho}\sin^2(\omega t - kz)$$

$$K_R = \frac{1}{4}\mathbf{w}\cdot\boldsymbol{\sigma}\psi = \frac{k^2\omega^2 Q_0^2}{8\rho}\cos^2(\omega t - kz)$$

$$i\partial_t \psi = \left(\gamma^5 \boldsymbol{\sigma}\cdot i\nabla\psi + \frac{1}{2}\breve{w}_0 \gamma^0 \psi\right) + \frac{1}{2}\mathbf{w}\cdot\boldsymbol{\sigma}\psi$$

$$\mathcal{E} = \mathbf{v}\cdot\mathbf{P} - 2U + 2K_R$$

RHS is NOT Hamiltonian (because nonlinear)

Conservation Law

Nonlinear Dirac equation:

$$\partial_t \psi = -c\,\gamma^5 \boldsymbol{\sigma}\cdot\nabla\psi - \mathbf{u}\cdot\nabla\psi - \frac{\mathrm{i}}{2}\left(\breve{w}_0\,\gamma^0\psi + \mathbf{w}\cdot\boldsymbol{\sigma}\psi\right)$$

Multiply $\psi^\dagger$ and add adjoint:

$$\partial_t\left(\psi^\dagger\psi\right) = -\nabla\cdot\left(\psi^\dagger c\,\gamma^5\boldsymbol{\sigma}\psi\right) - \mathbf{u}\cdot\nabla\left(\psi^\dagger\psi\right)$$

Magnitude $\psi^\dagger\psi$ is a conserved quantity.

Exclusion Principle & Potentials

- Wave superposition of "particles" A and B:

$$[\psi_A + \psi_B]^\dagger [\psi_A + \psi_B] = \psi_A^\dagger \psi_A + \psi_B^\dagger \psi_B \\ + \psi_A^\dagger \psi_B + \psi_B^\dagger \psi_A$$

- Interference terms cancel for "independent" particles $\Longrightarrow$ exclusion principle:

$$\psi_A^\dagger \psi_B + \psi_B^\dagger \psi_A = 0$$

- Potentials derive from phase shifts introduced to maintain zero interference.

$$\psi_A^\dagger e^{-i\varphi_A} e^{i\varphi_B} \psi_B + c.c. = 0$$

Exclusion Principle & Potentials

Neglect nonlinear terms.

Transformation of momentum operator:

$$\psi_A^\dagger(-i\nabla)\psi_A \rightarrow \psi_A^\dagger e^{-i\varphi_A}(-i\nabla)e^{i\varphi_A}\psi_A$$
$$= \psi_A^\dagger(-i\nabla + \nabla\varphi_A)\psi_A$$

Transformation of Hamiltonian:

$$\psi_A^\dagger(\hbar\partial_t + \mathrm{i}H)\psi_A \rightarrow \psi_A^\dagger e^{-i\varphi_A}(\hbar\partial_t + \mathrm{i}H)e^{i\varphi_A}\psi_A$$
$$= \psi_A^\dagger(\hbar\partial_t + \mathrm{i}H + \mathrm{i}\hbar\partial_t\varphi_A + \mathrm{i}\hbar c\gamma^5\boldsymbol{\sigma}\cdot\nabla\varphi_A)\psi_A$$
$$\equiv \psi_A^\dagger(\hbar\partial_t + \mathrm{i}H + \mathrm{i}e\Phi - \mathrm{i}\gamma^5\boldsymbol{\sigma}\cdot e\mathbf{A})\psi_A$$

Electromagnetic potentials:

$$e\mathbf{A} = -\hbar\nabla\varphi_A; \quad e\Phi = \hbar\partial_t\varphi_A$$

Exclusion Principle & Potentials

Change of wave momentum (force): $\frac{d}{dt}P_i =$

$$\psi_A^\dagger\Big((\partial_t\partial_i - \partial_i\partial_t)\varphi_A - c\gamma^5\sigma_j\big(\partial_i\partial_j - \partial_j\partial_i\big)\varphi_A\Big)\psi_A$$

Multivalued phase $\Longrightarrow$ Non-commuting!

Electromagnetic variables:

$$e\mathbf{A} = -\hbar\nabla\varphi_A; \quad e\Phi = \hbar\partial_t\varphi_A$$

$$\rho_e = e\psi_A^\dagger\psi_A; \quad \mathbf{J} = e\psi_A^\dagger c\gamma^5\boldsymbol{\sigma}\psi_A$$

$$\mathbf{E} = -\frac{\hbar}{e}\nabla(\partial_t\varphi_A) + \frac{\hbar}{e}\partial_t(\nabla\varphi_A) = -\nabla\Phi - \partial_t\mathbf{A}$$

$$B_k = \epsilon_{kij}\partial_i A_j = -\frac{\hbar}{e}\epsilon_{kij}\partial_i\partial_j\varphi_A$$

$$\frac{d}{dt}\mathbf{P} = \rho_e\mathbf{E} + \mathbf{J}\times\mathbf{B}$$

Exclusion Principle & Potentials

Stokes' Law: $\oint \mathbf{A} \cdot d\boldsymbol{\ell} = \iint (\nabla \times \mathbf{A}) \cdot d\mathbf{S}$

Macroscopic sytem:

Pure phase shift $\Rightarrow$ magnetic flux quantized:

$$\varphi_A = \left(m_\phi \phi - \omega t\right)$$

$$\mathbf{A} = -\frac{\hbar}{e} \nabla \varphi_A = -\frac{\hbar m_\phi}{er \ \sin\theta}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{\hbar m_\phi}{e} 2\pi \delta^{(\mathbf{2})}(x, y)$$

$$\oint \mathbf{A} \cdot d\boldsymbol{\ell} = -\frac{\hbar}{e} 2\pi m_\phi \xrightarrow{m_\phi = 1/2} \frac{h}{2e}$$

c.f. Hagen Kleinert, *Multivalued Fields in Condensed Matter, Electromagnetism, and Gravitation* (Chapter 4).

Exclusion Principle & Potentials

Radially weighted phase shift from particle:

$$\varphi_A = (m_\phi \phi - \omega t)\left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right)\frac{c}{\omega r}$$

$$e\mathbf{A} = \left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right)\frac{c}{\omega r}\left\{\left(\frac{m_\phi}{r\ \sin\theta}\right)\hat{\phi} - \frac{m_\phi \phi - \omega t}{r^2}\hat{r}\right\}$$

Neglect radial term (no flux). Note $\hbar\omega = m_e c^2$

$$e\mathbf{B} = \left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right)\frac{c}{\omega}\left(\frac{m_\phi}{r^3\ \sin\theta}\right)\hat{\boldsymbol{\theta}}$$

Same flux as e^- dipole moment: $\mu_0 M/2r$:

$$\oint \mathbf{A}\cdot d\boldsymbol{\ell} = -\frac{m_\phi e}{2\epsilon_0 \omega r} \xrightarrow{m_\phi = 1/2} \frac{\mu_0 \hbar e}{4 m_e r}$$

Exclusion Principle & Potentials

Radially weighted phase shift:

$$\varphi_A = (m_\phi \phi - \omega t)\left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right)\frac{c}{\omega r}$$

Phase velocity: $v_\phi = \frac{\omega}{m_\phi}\, r \sin\theta \ \rightarrow 2\omega\, r \sin\theta$

Electric field (Note $\partial_t \varphi_A = -\mathbf{v}\cdot\nabla\varphi_A$):

$$eE_i = \hbar\left(\partial_i v_j\right)\partial_j \varphi_A + \hbar v_j\left(\partial_i\,\partial_j - \partial_j\,\partial_i\right)\varphi_A$$

Only first term contributes ($v_\phi = 0$ at $r = 0$)

$$\mathbf{E} = \left(\frac{e}{4\pi\epsilon_0 r^2}\right)\hat{\mathbf{r}}$$

c.f. Herbert Jehle, Phys. Rev. 3(2):306-345, 1971

Matter & Antimatter

Assume vector spherical harmonic “particles”.

Bispinor angular quantum number is half of vector angular quantum number.

Vector: $\ell = 2N + 1 \Longrightarrow$ Bispinor: $\ell = \frac{2N+1}{2}$

Odd parity $\Longrightarrow$ Distinct mirror image $\Longrightarrow$ Fermion

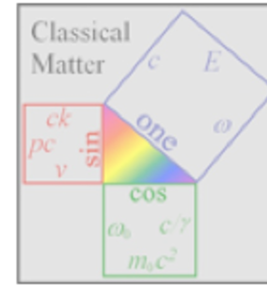
Vector: $\ell = 2N \Longrightarrow$ Bispinor: $\ell = N$

Even parity $\Longrightarrow$ Same mirror image $\Longrightarrow$ Boson

(bosons = antiparticles except W+/-)

Assumption valid except for W+/-.

Solid Aether Model



- Lorentz-invariant wave equation (SR).
- "Gravity" is wave refraction (GR).
- Spin is the angular momentum of the medium.
- Mass ⇒ spatial localization, velocity rotation.
- Spin & orbital angular momentum.
- Odd/even spherical harmonics: fermions/bosons
- Spatial reflection yields "antiparticles".
- Exclusion principle and interaction potentials.
- Interpretation of electromagnetism.
- Relation between magnetic flux & electric charge.
- Wave uncertainty principle.

Publications

- Torsion Waves in Three Dimensions: Quantum Mechanics With a Twist," **Found. Phys. Lett.** 15(1):71-83, Feb. 2002.
- "A Classical Dirac Bispinor Equation," in *Ether Space-time & Cosmology, vol. 3*, eds. M. C. Duffy & J. Levy, (Aperion, Montreal, 2009).
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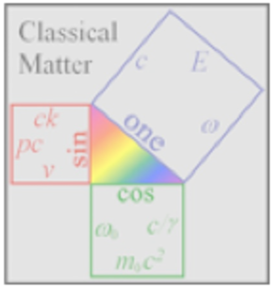

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