Matter Waves and Relativity

Name_____________ Partner ______________ Partner ______________

When waves propagate along a string, pieces of the string move locally back and forth with arbitrary speed determined by the amplitude and frequency of the wave, while the wave energy propagates along the length of the string at a fixed wave speed determined by the tension and mass density of the string. When you throw a ball, or a meteor zooms through space, can the object have arbitrary speed, or does it have a fixed speed like a wave?

The surprising answer is that matter propagates through space like a wave! Although the speed appears variable, one can model the motion of matter through space as a wave propagating at the speed of light in circular, spiral, or cycloidal paths (a cycloidal path is like the motion of a point on a rolling bicycle tire). You could even combine two waves circulating in opposite directions to create a standing wave. We will investigate this model of matter in this lab, looking at one of the waves on the circle.

Although not mainstream, the wave model of matter has some experimental support. Light waves with frequency f come in packets (photons) of energy:

$$E = hf$$

where the number $h = 6.6\times10^{-34}$ J·s is called "Planck's constant".
Albert Einstein's analysis of relativistic motion implied that energy is related to rest mass (m_0). For a particle at rest:

$$E=m_0c^2$$

If rest energy arises from light-like waves propagating in circles, we can equate the two expressions for energy to obtain the frequency (f_0) of a particle at rest:

$$f_0 = \frac{m_0c^2}{h}$$

For an electron, this frequency is $f_0 = 0.77634 \times 10^{21}$ Hz. Scientists recently reported evidence of this oscillation [M. Gouanère *et. al.*, A Search for the de Broglie Particle Internal Clock by Means of Electron Channeling, *Foundations of Physics* **38**: 659-664 (2008).].

Materials: Cylindrical pen, string, tape

Objectives:

- To understand the wave model of matter and special relativity

Section A: Relativistic time dilation

Cut a piece of string and tape (or otherwise attach) one end of it to one end of the pen. Cut the length of the string so that it wraps around the pen exactly four times. This represents the distance (ct_0) light travels in 4 "ticks" of a stationary clock (t_0=4 ticks). This clock would tick very fast since light travels about one foot each nanosecond! The pen with thread represents a "light clock" which ticks once each time light travels around the circumference.

Length of string: L_0 = __________________________ cm (to nearest 0.02 cm)

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Wrap the string tightly around the cylinder so that it makes exactly 4,3,2 or 1 complete turns. The number of turns represents the number of "ticks" made by a moving clock. The distance (L) between the ends represents the distance a particle would travel in 4 "ticks" of a stationary clock. Measure the distance (L) and complete the following table:

t = No. turns	$(t_0/t)=4/t$	L	$(v/c)=L/L_0$	Gamma $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$
4	1.00	0	0	1
3	1.33			
2	2.00			
1	4.00			
0	∞		1	∞

Compare the computed values of gamma with the time ratios (t_0/t). Because of the way gamma is calculated, as v/c approaches 1.0 the uncertainty in gamma increases. The experimental uncertainties are about 10% for $t_0/t = 1.33$; 20% for $t_0/t = 2.0$; and 80% for $t_0/t = 4.0$. **Are your computed values of gamma equal to the time ratios t_0/t within the measurement errors)?**

Does a moving clock tick faster or slower than a stationary clock?

Relativistic time dilation of matter has been carefully measured by flying atomic clocks in airplanes. It really happens! Also, when particles are accelerated to high energies, their speed approaches but never quite equals the speed of light. **Use the model of matter as circulating waves to explain why:**

Now let's try to model the actual wave pattern. Consider the following picture of wave propagation with the circular direction "unrolled" top-to-bottom along the page:

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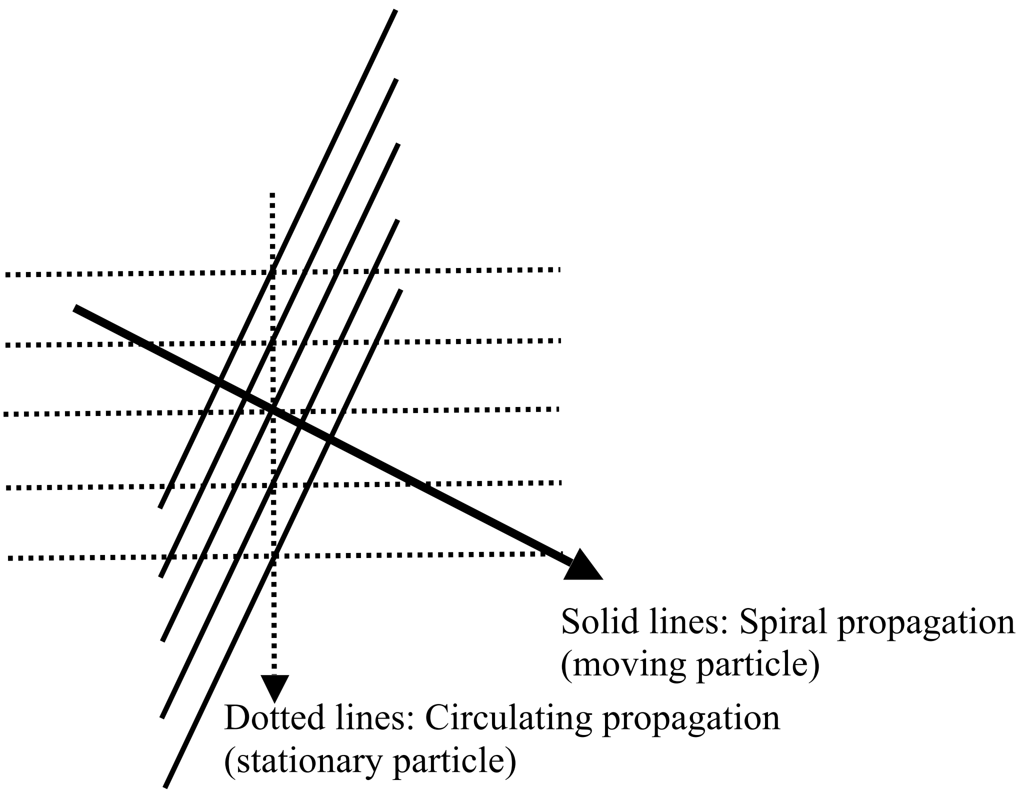

Figure 1: Wave fronts for circular and spiral propagation with the circular direction unrolled on the page from top-to-bottom. Try rolling up the page to construct the circular patterns.

Notice that the spacing along the circulating direction is the same for both the stationary and moving particles. This spacing determines the rest mass. The wave propagates perpendicular to the wave crests. The spiral propagating wave has smaller actual wavelength and therefore higher wave frequency.

Measure the wavelength of:
(a) the stationary particle (distance between dotted lines) ___________ cm

(b) the moving particle (distance between solid lines) ___________ cm

Recall that (wave frequency) = (wave speed) / (wavelength). Compute the ratio of frequencies:

$$\frac{(\text{moving frequency})}{(\text{stationary frequency})} = \frac{(\text{stationary wavelength})}{(\text{moving wavelength})} = \gamma = \underline{\hspace{3cm}}$$

Write an equation relating the wave frequency (*f*) of moving particle with the wave frequency (f_0) of a stationary particle:

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Does a moving particle have higher or lower wave frequency than a stationary particle?

If you said "lower", then get help from your instructor. Earlier you showed that a moving clock ticks slower than a stationary clock (lower ticking rate). Now you have shown that a moving particle (a type of clock?) has higher frequency than a stationary clock. **Are these two results compatible?**

(the answer is "yes")
What does the frequency of a wave represent?

What does the ticking rate of the particle clock represent?

Explain how the frequency of the wave can increase while the rate of clock ticking decreases:

Notice in the picture that the horizontal extent of the waves also changes with motion. Check that the dotted lines are the same length as the solid lines in the figure above (if not, then extend one set to make them equal lengths).

Measure the horizontal length (left to right, not diagonal) of

(a) the stationary wave (dotted lines) _______________cm

(b) the moving wave (solid lines)_______________cm

Is the moving wave longer or shorter than the stationary wave?

The shortening of length of moving objects is called "length contraction".
Compute the ratio of lengths:

$$\frac{(\text{stationary length})}{(\text{moving length})} = \gamma = \_\_\_\_\_\_\_\_\_\_\_\_$$

Is this value of γ approximately equal to the value computed earlier from wavelength measurements?

Write an equation relating the length (L) of moving object with the length (L_0) of a stationary object:

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Exercises:

1. Two twins live on the same planet. One hops in a spaceship and takes a journey traveling close to the speed of light. When the traveling twin returns to greet the other, which twin will be older? If the stationary twin waited 6 years for the other, and the traveling twin had an average gamma of 1.2, what is their difference in age?

2. A docked spaceship has a length of 20 meters. When in flight, how fast would it have to move in order for a distant observer to see it be instantaneously eclipsed by an 18-meter diameter spherical asteroid? (Neglect motion of the asteroid and assume near-parallel rays of light from the edges of the asteroid to the observer.)

One final note: We have examined the difference between "stationary" and "moving" particles. How do we know which is which? One of the miraculous properties of nature is that any non-accelerating observer may assume that he or she is stationary. It is impossible to determine absolute motion! This is a consequence of a mathematical property of wave equations called "Lorentz Invariance".